1 Measurement and Probability

Some games are based on skill. Some are based on chance. Many contests, lotteries, and games offer the chance of winning a prize. Prizes range from a free coffee to a car to a million dollars. But what is the probability of winning?

1. What is the difference between games based on skill and games based on chance?

2. Name one game of chance that you have played.

3. Describe what makes it a game of chance.

Key Words

accuracy
precision
tolerance
probability
odds
tree diagram
theoretical probability
experimental probability

Career Link

A number of jobs depend on the weather. Brandon owns a fishing boat. Before he takes his boat out each day, he checks the weather forecast. He needs to be aware of the probability of bad weather.
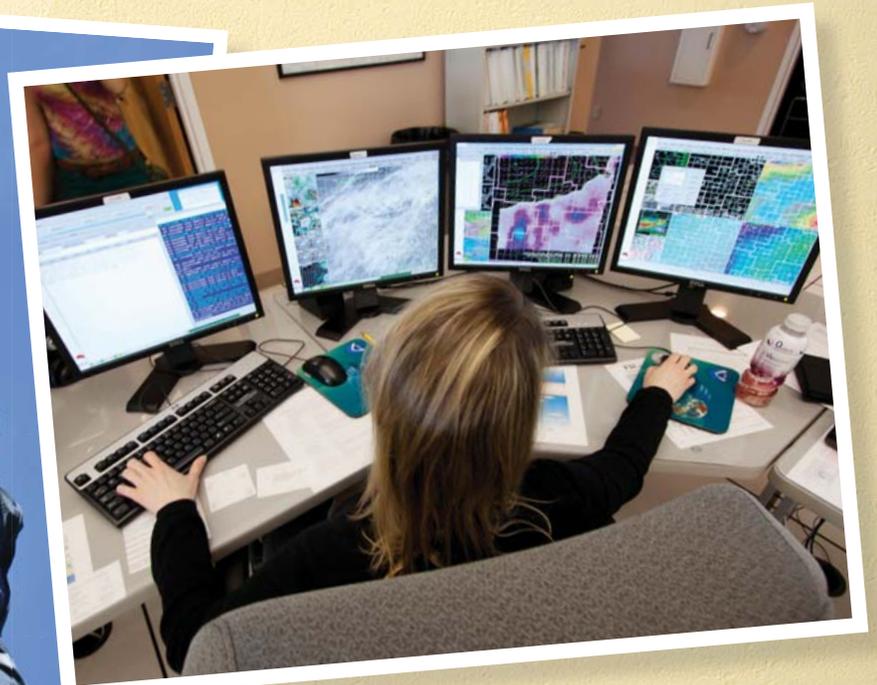
Get Ready

Fractions, Decimals, and Percents

1. a) What amount of money is shown? Express your answer in two ways:

■¢ $■

b) Write this amount as a fraction of a dollar. Express the fraction in two ways.

c) Express this amount as a percent of a dollar.

2. Write each fraction in lowest terms.

a) $\frac{2}{8}$ **b)** $\frac{3}{9}$ **c)** $\frac{14}{28}$

d) $\frac{20}{24}$ **e)** $\frac{9}{33}$ **f)** $\frac{8}{8}$

3. This picture of a measuring tape shows 1 foot.

a) How many inches is 1 foot?

b) How many inches is $\frac{1}{2}$ foot?

c) How many inches is $\frac{1}{4}$ foot?

d) What percent of 1 foot is 6 inches?

4. What is the tax on a purchase of each amount? Do not use a calculator.

What is the percent sales tax where you live?

a) $1   **b)** $2

c) $3   **d)** $10

e) $20   **f)** $30

g) $100   **h)** $200

i) $300   **j)** $333

5. Copy the table. Determine the missing values.

	Fraction	Decimal	Percent
a)	$\frac{1}{2}$		
b)	$\frac{1}{10}$		
c)		0.35	
d)		0.07	
e)			9%
f)			90%

6. Determine each answer without using a calculator or a calendar.

a) How many weeks are in one year?

b) How many weeks are in a $\frac{1}{2}$ year?

c) How many seasons are there?

d) How many weeks are in one season?

**e)** Express your answer to part d) as a fraction of a year, in lowest terms.

Interpreting Graphs

7. Diane moved into her own apartment a month ago. The graph shows her living expenses for one month.

Living Expenses for One Month

- rent
- groceries
- car
- Internet
- phone
- entertainment
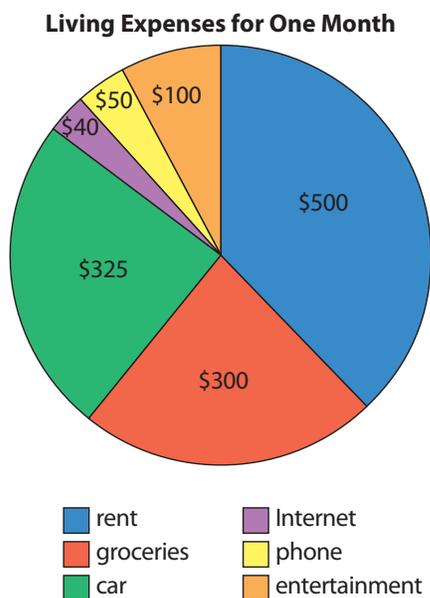
a) Approximately what fraction of her expenses did Diane spend on rent?

b) Approximately what fraction of her expenses did she spend on her car?

c) What two expenses make up about half of her total expenses?

d) What is the total amount of money that Diane spent last month?

e) Express each of Diane's expenses as a percent of the total.

8. The graph shows the number of music downloads over one week.
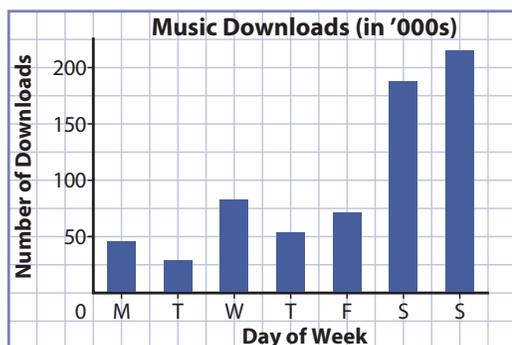
a) Explain the meaning of "in '000s."

b) Estimate the number of downloads for each day of the week.

c) Use your answer to part b) to estimate the total number of downloads in one week.

d) Explain the large number of downloads on Saturday and Sunday.

Exponents

9. Express each power as repeated multiplication.

  a) 4^3 **b)** 2^4

  c) 6^2 **d)** 10^4

  e) 3^4 **f)** 5^3

  g) 2^6 **h)** 10^6

10. Evaluate each part of #9 without using a calculator.

1.1 Accuracy and Precision

Focus On . . .

- understanding the difference between accuracy and precision
- determining the importance of degrees of accuracy and precision
- calculating maximum and minimum values, using a given degree of tolerance

accuracy

- the degree to which a measurement is measured and reported correctly

precision

- the degree of exactness to which a measurement is expressed
- the precision of a measurement depends on the scale of the instrument used

You could say that the angle shown measures 48.5°, 49°, or 50°. All of these measurements have **accuracy,** *they just vary in their* **precision.** *If you say that the angle is about 130°, that is not accurate—you are reading the protractor incorrectly.*
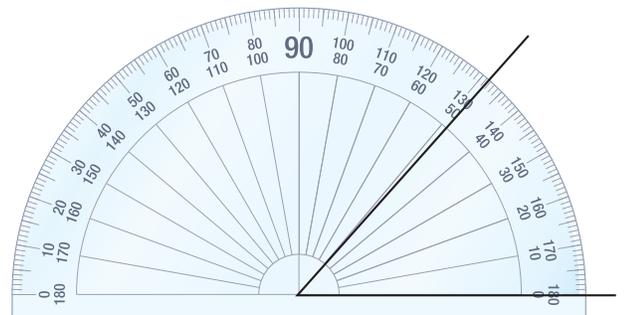
Explore Accuracy and Precision

One very common measuring error is to measure something properly but state the measurement in the wrong units. In the photo above, the width of the door is about 60 cm. It would be inaccurate to say that it is about 60 feet wide or about 60 inches wide.

Materials

• ruler, metre stick, and/or measuring tape

1. As a class, choose six items in your classroom to measure.

2. Work with a partner and measure the length, width, and height of each item. For some items, you make not be able to take all three measurements.

3. Make a table like the one shown to record your data.

Item	Length	Width	Height

4. Compare your measurements with those of other teams.
   a) Which measurements were the same as the measurements done by other teams?
   b) Which measurements were close to the measurements done by other teams?
   c) Which measurements were quite different from the measurements done by other teams?
   d) Which measurements were in different units from what other teams used?

5. **Reflect**
   a) Explain how two teams can measure the same item yet report slightly different measurements.
   b) Explain how two teams can measure the same item yet report significantly different measurements.

6. **Extend Your Understanding** Find a team that had different measurements from your team. Compare how each team measured each item. What did your team do differently from the other team that resulted in different measurements?

On the Job 1

Determining Accuracy

Vic and his son are going to pour a concrete pad before building a shed. They want the pad to be 12 ft long, 8 ft wide, and 4 in. thick.

Vic's son wants to calculate the volume of the concrete needed. He knows that
volume = length × width × height (or thickness).

$V = lwh$
$V = 12 \times 8 \times 4$
$V = 392$

> Height can also be thickness.

Vic's son says, "We need 392 cubic feet of concrete."

Vic says, "No way. We need only 32 cubic feet."

a) Whose calculation is accurate? Explain.

b) Is it important for this calculation to be accurate? Explain.

Solution

a) The son's calculation was $12 \times 8 \times 4$, which is 392. However, the 12 and the 8 are measurements in feet, while the 4 is a measurement in inches. To have a volume in cubic feet, all of the measurements must be in feet. The son's calculation was not accurate. The measurements were right, but they were not used properly.

Vic shows his son how to do the calculation:

$V = 12 \times 8 \times \frac{1}{3}$
$V = 32$

> 4 in. is $\frac{1}{3}$ of a foot.

Vic's calculation is accurate. They need 32 ft³ of concrete.

> $12 \times 8 = 96$
> $\frac{1}{3}$ of 96 is 32.

b) Yes, it is important for the calculation to be accurate. If the calculated amount of concrete needed is too low, they will not have enough to complete that job. If it is too high, they will have bought too much concrete mix, and may not be able to return it for a refund.

Your Turn

Calculate the volume of concrete needed for a pad that measures 20 ft by 10 ft by 6 in.

Try It

**1.** Determine whether each angle measurement is accurate.

a)

∠ABC = 42°

b)
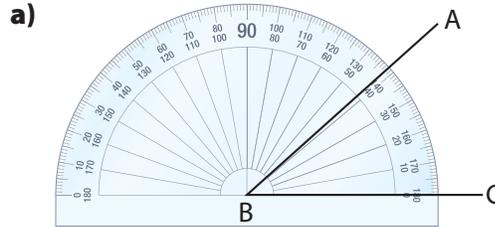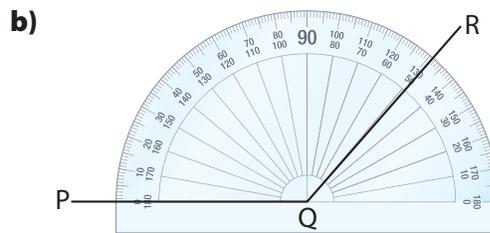
∠PQR = 59°

2. Explain why saying that ∠PQR in #1b) equals 130° can be considered accurate.

3. Which is an accurate location of the arrow?

A 7.5 cm   **B** 9 cm   **C** $7\frac{1}{2}$ in.
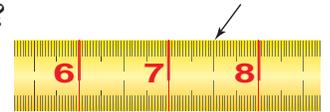
4. Explain why the other two given measurements in #3 are not accurate.

Apply It

5. Joanne is driving from Edmonton, AB, to Saskatoon, SK. What is an accurate estimate of the distance? The scale of the map is 1 cm : 100 km.
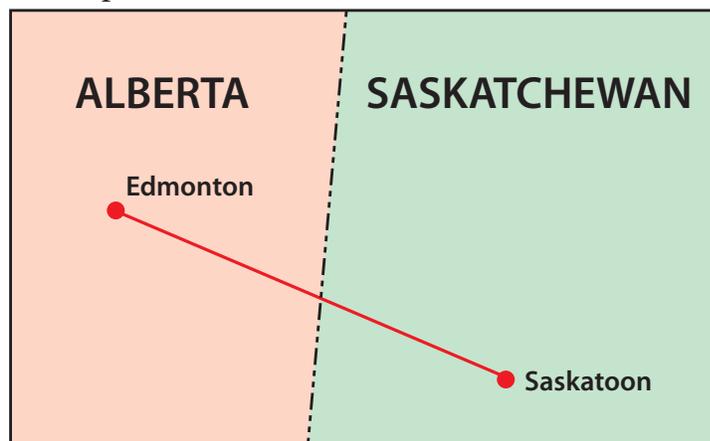
6. Joanne says that the distance between Edmonton, AB, and Saskatoon, SK, is about 5.3 hours. **Note:** Refer to #5.

   a) Explain why this statement is not accurate.

   b) What do you think Joanne means?

   c) Do you think it is important for Joanne to be accurate in this case? Explain.

7. Chad is going to stain and varnish a table. He says that the diameter of the tabletop is 46 inches. Explain why his measurement is not accurate.

46″
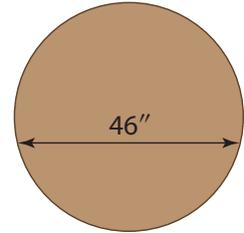
8. Jack estimates that the width of his property is about 60 to 70 metres. He has a measuring wheel and a 10-metre measuring tape. Which measuring tool will likely give Jack the most precise measurement? Explain your reasoning.

9. We often use inaccuracy to make a point. Explain the everyday meaning of each inaccurate statement.

   a) It is a million degrees outside.

   b) It's going to take a century to clean my room.

   c) That thing weighs a ton.

   d) His answer was a mile off.

On the Job 2

Determining Degree of Precision

Greg is redecorating a bedroom in his house. He knows that his measurements need to be accurate, but the degree of precision required depends on the job.

a) Greg wants to determine how much paint to buy. The dimensions of the room are $12'\,8\frac{1}{2}''$ by $10'\,10''$. The ceiling is $7'\,10\frac{1}{2}''$ high. What degree of precision does he need for the room's measurements? How much paint should he buy?
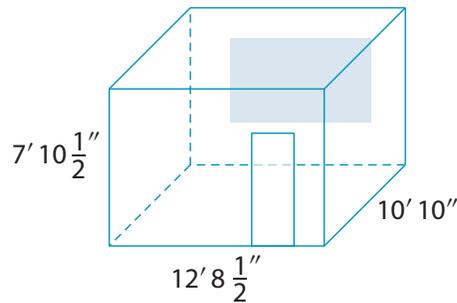
b) Greg is also replacing the door and window casing. The door frame measures $80''$ by $30''$. The door casing will be installed so that about $\frac{1}{8}''$ of the door frame is exposed.

Greg needs to cut the pieces of casing. What degree of precision does he need for the measurements? How long should he cut the pieces of casing?

Solution

a) When determining how much paint to buy, the measurements do not need to be precise.

Greg rounds the room dimensions to 13′ long by 11′ wide by 8′ high.

Area of longer walls $= (13 \times 8) \times 2$
$$= 208$$

Area of shorter walls $= (11 \times 8) \times 2$
$$= 176$$

Greg will put two coats of paint on the walls.

Total area $= (208 + 176) \times 2$
$$= 384 \times 2$$
$$= 768$$

Greg does not subtract the area of the doors and windows. Instead, he estimates how much less paint he will need. Greg decides he needs to buy enough paint to cover about 750 square feet.

b) When cutting door casing, the measurements need to be precise.

For the two vertical pieces of casing, Greg adds $\frac{1}{8}''$ to the 80″ height of the door frame:

$$80 + \frac{1}{8} = 80\frac{1}{8}$$

He will cut this piece of casing so that it is $80\frac{1}{8}''$ long.

For the horizontal piece of casing across the top of the door, Greg adds $\frac{1}{8}''$ to either end of the 30″ width of the door frame:

$$30 + \frac{1}{8} + \frac{1}{8} = 30\frac{1}{4}$$

Greg will cut this piece of casing so that the distance between the endpoints touching the door frame is $30\frac{1}{4}''$.

Your Turn

a) A second bedroom in Greg's house has dimensions that are 10 ft 4 in. by 13 ft 10 in. The ceiling is 7 ft $10\frac{1}{2}$ in. high. He wants to paint the bedroom. How much paint should he buy?

b) A closet door in the bedroom is 80 inches high and 24 inches wide. Determine the length he should cut the horizontal piece of casing that goes across the top of the closet door frame.

Tools of the Trade

Different measuring tools have different degrees of precision. The smallest measurement shown on a measuring tape is 1 mm. The smallest measurement shown on a Vernier calliper is 0.1 mm. Vernier callipers are used in mechanical engineering, metalworking, woodworking, science, and medicine. For more information on Vernier callipers, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Try It

1. Measure the lines. State the length of each line to the nearest half inch.

a)
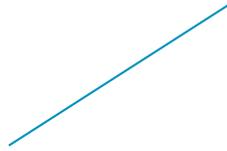
b)

c)

2. Remeasure the lines from #1.

    a) State the length of each line to the nearest sixteenth of an inch.

    b) State the length of each line to the nearest centimetre.

    c) State the length of each line to the nearest tenth of a centimetre.

3. Measure the following angles. State each measure to the nearest 5°.

a)
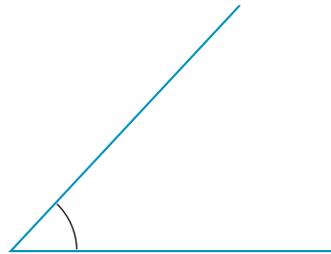
b)

c)
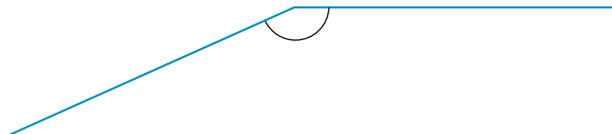
4. Remeasure the angles from #3. State each measure to the nearest degree.

Apply It

5. Brett weighs himself in the morning on his bathroom scale. It shows 180 lb. When he gets to the gym, Brett weighs himself again. The scale at the gym says 181.6 lb.

   a) What units is each scale using to display mass? Explain how you know.

   b) Which scale is more precise? Explain.

   c) Explain how both scales could be accurate.

6. a) What precision is usually used when gas stations display the price of a litre of gasoline?

Self Service

141.1

   b) Is it possible to pay $1.411 for exactly 1 litre of fuel? Explain.

   c) At the price shown, how much would you pay for exactly 1 litre of gasoline?

   d) How much would you pay for exactly 2 litres?

   e) If you were to buy exactly 1 litre of gas each day for 10 consecutive days, how much would you pay in total, assuming the price stays the same?

   f) How much would you pay for a single purchase of 10 litres of gas?

   g) If you make a single purchase of exactly 10 litres of gas, what is the average price per litre?

7. Marci is $5' \, 6\frac{3}{4}''$ tall.

   a) Round Marci's height to the nearest inch.

   b) Round Marci's height to the nearest foot.

   c) Do you think it is accurate for Marci to tell people that her height is the answer to part a)? Explain.

   d) Do you think it is accurate for Marci to tell people that her height is the answer to part b)? Explain.

On the Job 3

Tolerance

A clothing manufacturer allows for a certain **tolerance** when making their products. Men's pants are sold in whole-inch sizes by their waist measurement and, often, by their inseam measurement as well. The company allows for a tolerance of $\pm\frac{1}{4}''$ when labelling their products.

waist measurement

inseam measurement

F.Y.I.

$\pm\frac{1}{4}''$ is read, "plus or minus one-quarter inch."

a) What is the tolerance for the waist measurement of a pair of pants?

b) What are the maximum and minimum allowable waist measurements that can be labelled as 32-inch waist pants?

Solution

a) The tolerance in the waist measurement is $\frac{1}{4}''$.

b) A pair of pants that has a waist measurement from $31\frac{3}{4}''$ to $32\frac{1}{4}''$ can be labelled as 32″. For example, pants with a waist measurement that is greater than $32\frac{1}{4}''$ will have to be resewn and pants with a waist measurement of $32\frac{3}{4}''$ will be labelled as 33″.

Your Turn

The same manufacturer allows a tolerance for the neck measurement of a men's shirt of $\pm\frac{1}{8}''$. Their men's shirts are sold in $\frac{1}{2}$-inch increments of the neck measurement, for example, $14\frac{1}{2}''$, 15″, $15\frac{1}{2}''$, 16″, and so on.

neck measurement

a) What is the tolerance for the neck measurement of a men's shirt?

b) What are the maximum and minimum allowable measurements that can be sold as a men's shirt with a 15-inch neck?

Try It

1. Determine the maximum and minimum allowable measurements.

   a) $22'' \pm \dfrac{1}{4}''$ b) $45\,°C \pm 1°$ c) $350\,°F \pm 10°$

   d) $1\,m \pm 1\,cm$ e) $1\,m \pm 1\,mm$ f) $5\,lb \pm 0.2\,lb$

2. For each part of #1, what is the tolerance?

3. Determine the tolerance if the maximum and minimum allowable measurements for a 100-g product are 95 g and 105 g.

Apply It

4. A meat thermometer is accurate to $\pm 2°$. What range of temperatures could the interior temperature of the burger be?

5. A machine shop that manufactures nuts and bolts allows a tolerance of ± 0.01 mm.

   a) What are the maximum and minimum diameters of a 6-mm bolt?

   b) The corresponding nut has an inside diameter of 6.1 mm, ± 0.01 mm. What are the maximum and minimum allowable inside diameters of the nut?

   c) What is the greatest possible difference between the diameter of the bolt and the nut? Explain.

   d) What is the least possible difference between the diameter of the bolt and the nut?

   e) Explain why the nut cannot be manufactured with an inside diameter of 6 mm.

6. Most breakfast cereals are sold by mass, not by volume. A 350-g package is allowed to have a tolerance of ± 3 g. What is the acceptable range of masses for a cereal package?

7. A kitchen scale is accurate to ± 10 g. Jodie places a 350-g package of breakfast cereal on her scale. The scale reads 345 g.

   a) Can you tell if Jodie's scale is accurate? Explain.

   b) Is her scale likely to be as precise as the machines in the cereal factory? Explain.

8. The information on a package of carrot seeds says that the carrots, under normal growing conditions, will grow to a length of 6 inches to 8 inches.

   a) What do you think is the average length of the carrots? Explain.

   b) Express the likely length of each carrot using the $\pm$ symbol.
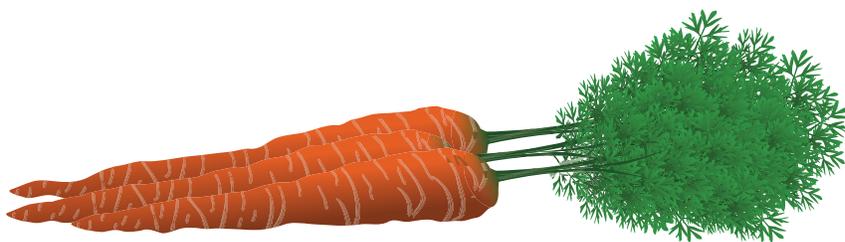
1. At its farthest, Earth is 152 098 232 km from the sun.

   a) What degree of precision do you think is used to state this distance? Explain.

   b) Look at the diagram. What degree of precision do you think is used to state the distance between Earth and the sun?

   c) What is the difference between the distance in part a) and the distance in the diagram?

   d) Do you think it is accurate to state the value in the diagram as the distance from Earth to the sun? Explain.
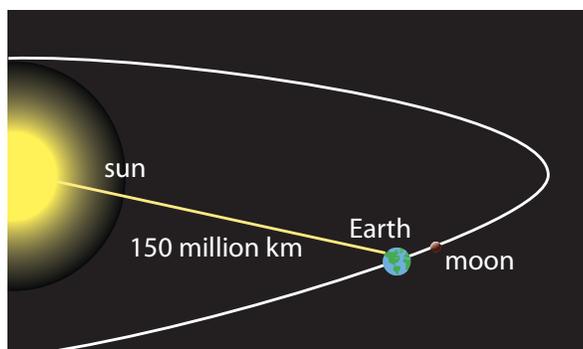
2. Raheem needs to give his dog 3 mL of medication twice a day. Raheem does not have metric measuring spoons, but knows that 1 teaspoon equals about 5 mL. He thinks he could estimate 3 mL by measuring out just over half a teaspoon of medication. Is he using an acceptable degree of precision? Explain.

3. Marg is a baker. Her friend, Jamie, passes along the recipe for a cake that Jamie says is delicious. The recipe includes "2 dashes of baking soda" and "2 pinches of salt."

  a) Marg thinks that 1 dash is about $\frac{1}{8}$ teaspoon and 1 pinch is about $\frac{1}{16}$ teaspoon. How much baking soda and how much salt could the recipe be calling for?

  b) Should Marg make the cake using the amounts in part a)? Explain.

4. Sarah needs to buy material to sew a duvet cover. The duvet itself is 90 inches by 8 feet. She multiplies and decides to buy 720 square inches of material.

  a) Is her calculation accurate?

  b) Is it important for this calculation to be accurate? Explain.

Discuss It

5. The conductors of an opinion poll claim that their results are accurate to within 5%. A new flavour of soft drink is liked by 52% of the people surveyed. Explain why it is possible that the majority of the population may dislike the new soft drink.

6. Is it possible for a measurement to be precise but not accurate? Give an example.

7. Is it possible for a measurement to be accurate but not precise? Give an example.

8. Steve uses the terms *accuracy* and *precision* interchangeably. In your own words, explain the difference between accuracy and precision to Steve.

9. Research the precision of at least three measuring tools used in the trades. To what degree of precision does each tool measure? Give examples of jobs that require the use of each tool.

10. You are asked to measure to the nearest tenth of an inch but your ruler has intervals of sixteenths. How accurate will your measurement be?

Probability and Odds

Focus On . . .

- determining the probability of an event
- expressing a probability as a fraction, decimal, and percent, and in words
- determining the odds for and against an event

probability

- the mathematical likelihood of something happening
- a ratio that compares the number of possible successful outcomes to the total number of possible outcomes

Materials

- 1 die
- grid paper
- ruler

Have you ever played Yahtzee™? Games like this involve rolling dice to try to get certain combinations of numbers. Does a player have a greater **probability** *of rolling some combinations than others?*

Explore Probability

1. Suppose you roll one die.

a) What are all of the possible rolls you can get?

b) Suppose you roll a die 50 times. Predict the number of times you will roll a 3.

2. a) Copy the table shown in your notebook. In the first column, record all of the possible rolls. Refer to your answer in step 1a).

Number on Die	Tally	Number of Times Rolled

b) Roll one die exactly 50 times. In the second column, keep a tally of the number of times you get each roll.

c) In the third column, write down the total for each tally.

d) On grid paper, create a bar graph showing your results. Label the *x*-axis "Number on Die." Label the *y*-axis "Number of Times Rolled." Include a title for the graph.

3. Reflect

a) Look back at your prediction in step 1b). Did you roll a 3 the number of times that you predicted? If not, how close were your results to your prediction?

b) Look at the results of other students. Did any of them roll a 3 the number of times that you predicted? How close were their results to your prediction?

4. Extend Your Understanding

Look at the graphs created by other students.

a) Is there a pattern to the general shape of each graph? If so, what is it?

b) What are the reasons for this pattern?

c) Do you think the pattern would change if you rolled two dice? Explain the reasoning behind your prediction.

Calculate Probability

A fisheries officer needs to measure the length of three different kinds of fish: pike, trout, and whitefish. The lake has been stocked with 250 fish.

- 25 fish are pike.
- 75 fish are trout.
- 100 fish are whitefish.

The officer catches the first fish to be measured. What is the probability that the fish is

a) a pike?

b) a trout?

c) a whitefish?

d) any one of these three kinds of fish?

Express each answer in four ways: fraction, decimal, percent, and words.

Solution

a) There are 25 pike in a total of 250 fish. The first fish caught could be any one of the 25 pike.

Fraction: $\dfrac{25}{250} = \dfrac{1}{10}$

Write the fraction in lowest terms.

Decimal: $1 \div 10 = 0.1$

Percent: [C] [1] [÷] [10] [2nd] [%] [=] 10

Experiment with your calculator to find out what keystrokes give you the answer.

The probability is 10%.

Words: The probability of the fish being a pike is 1 in 10.

b) There are 75 trout in a total of 250 fish.

Fraction: $\dfrac{75}{250} = \dfrac{3}{10}$

Decimal: $3 \div 10 = 0.3$

Percent: [C] [3] [÷] [10] [2nd] [%] [=] 30

The probability is 30%.

Words: The probability of the fish being a trout is 3 in 10.

c) There are 100 whitefish in a total of 250 fish.

Fraction: $\dfrac{100}{250} = \dfrac{2}{5}$

Decimal: $2 \div 5 = 0.4$

Percent: [C] [2] [÷] [5] [2nd] [%] [=] 40

The probability is 40%.

Words: The probability of the fish being a whitefish is 2 in 5.

d) The total number of pike, trout, and whitefish is
$25 + 75 + 100 = 200$.

Fraction: $\dfrac{200}{250} = \dfrac{4}{5}$

Decimal: $4 \div 5 = 0.8$

Percent: [C] [4] [÷] [5] [2nd] [%] [=] 80

The probability is 80%.

Words: The probability that the fish is any one of these three kinds is 4 in 5.

Your Turn

You are fishing in another lake that has a stocked population of 360 fish.
- 162 fish are whitefish.
- 108 fish are trout.

What is the probability that one of the fish you catch is
a) either of these two kinds of fish?
b) a trout?

Try It

1. a) What are all of the possible combinations for rolling two dice? Copy and complete the table shown. The first two rows have been done for you.

This roll can be expressed as (1, 2).

This roll can be expressed as (2, 1).

Total of Two Dice	Possible Outcomes	Number of Combinations
2	(1, 1)	1
3	(1, 2) (2, 1)	2
4		

b) What is the total number of possible outcomes from rolling two dice?

c) Which total has the greatest likelihood of occurring?

d) What is the probability of this total occurring? Express your answer as a fraction and as a percent.

2. a) On grid paper, create a bar graph showing the data from #1. Label the x-axis "Total of Two Dice." Label the y-axis "Number of Combinations." Include a title for the graph.

b) Compare the general shape of this graph to the prediction you made in step 4c) of Explore Probability on page 21. Does the graph support your prediction?

3. a) What is the probability of rolling a total of 9 with two dice? Express your answer as a fraction.

b) Rolling a sum of 3 has the same probability as rolling what other number?

4. A standard deck has 52 cards. If you choose one card, what is the probability of choosing

a) a red card?   **b)** a club?

c) a queen?

Express each answer as a fraction and as a percent.

Apply It

5. Part of a meteorologist's job is to predict the probability of precipitation (POP). In Gander, NL, one day the POP is 60%. What is this probability in words, as a fraction, and as a decimal?

6. A new medication for migraines has come on the market.

a) The medication causes serious side effects in 2 of 100 patients. Express this probability as a fraction, a decimal, and a percent.

b) The medication is 90% effective. Express this probability in words, as a fraction, and as a decimal.

c) Melanie has a migraine once a year. Should she take the medication? Explain why or why not.

d) Ian has a migraine twice a month. Should he take the medication? Explain why or why not.
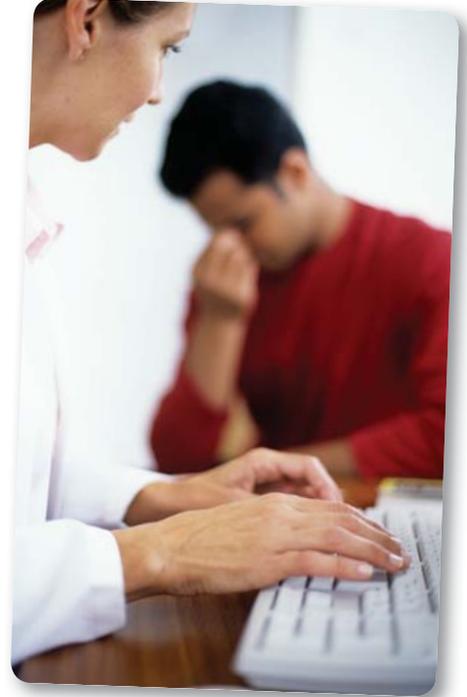
7. Manufacturers need to determine the probability of their products failing. The chart shows a standard way of ranking probabilities of failure.

a) A car part has a failure probability of 1 in 2000. What is the failure ranking? What is this probability as a decimal and a fraction?

b) An electronic part in a television has a failure probability of 12.5%. What is the failure ranking? What is this probability in words?

Probability of Failure	Ranking
more than 1 in 2	Very high: failure almost inevitable
1 in 3	
1 in 8	High: repeated failures
1 in 20	
1 in 80	Moderate: occasional failures
1 in 400	
1 in 2000	
1 in 15 000	Low: relatively few failures
1 in 150 000	
fewer than 1 in 1 500 000	Remote: failure is unlikely

c) What do you think should be the highest acceptable failure ranking for a car? for a television? Explain.

On the Job 2

Calculate Odds

For the grand opening of her coffee shop, Maria is having a promotion. Each customer who makes a purchase gets a scratch-and-win card. Half of the cards are winners. Suppose a customer has three scratch-and-win cards.

a) What is the probability of all three cards being winners?

b) What are the **odds** of all three cards being winners?

Solution

a) Use a **tree diagram** to determine all of the possible outcomes from scratching the three cards.

With the first card, there is a 1 in 2 chance of winning.

With the second card, there is a 1 in 4 chance of both cards being winners.

With the third card, there is a 1 in 8 chance of all three cards being winners.

W = Winner
T = Try Again

There is 1 successful outcome in a total of 8 possible outcomes.

The probability of getting three winners is $\frac{1}{8}$.

b) There is 1 successful outcome for every 7 unsuccessful outcomes.

The odds of getting three winners are 1 : 7.

Your Turn

a) What is the probability of scratching two cards and getting two winners?

b) What are the odds of scratching two cards and getting two winners?
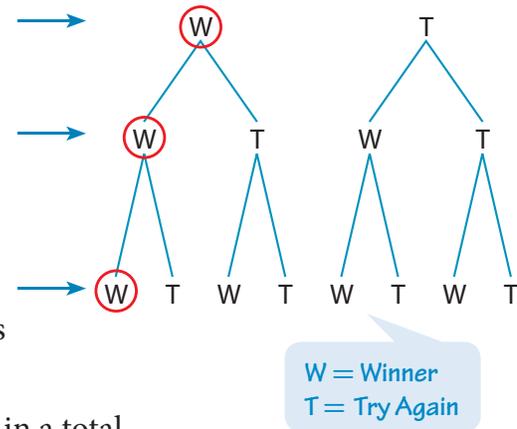
Try It

1. A standard deck has 52 cards. What are the odds of choosing

   a) the 7 of spades? **b)** a queen?

   c) a club? **d)** a red card?

2. You have one six-sided die. What are the odds of rolling

   a) a 2?

   b) a 5 or a 6?

   c) an odd number?

   d) any number but a 3?

3. A bag holds 10 coloured marbles. Of the marbles, 6 are red, 3 are blue, and 1 is green. What are the odds of

   a) selecting the green marble?

   b) selecting a blue or a green marble?

   c) selecting a red marble?

   d) not selecting a blue marble?

4. Rhys flips two coins at the same time.

   a) What are the odds that he gets one head and one tail?

   b) What is the probability that he gets one head and one tail?
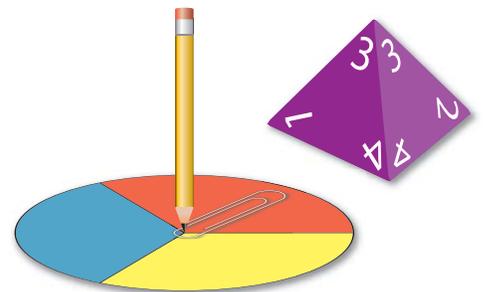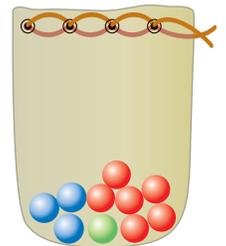
Apply It

F.Y.I.

On a four-sided die, the number that is rolled is the number at the top of the die. For example, in the four-sided die shown, the number rolled is 3.

5. A board game involves rolling a four-sided die, and then spinning a spinner with three equal sectors, coloured red, yellow, and blue.

   a) Construct a tree diagram to display all of the outcomes.

   b) How many total outcomes are there?

   c) What is the probability of each outcome?

   d) What are the odds of each outcome?

6. Sixteen contractors are chosen to present their proposals for building a new civic centre. The order of the presentations is determined by a random draw. The first name drawn presents first, the second one drawn presents second, and so on.

   a) What are the odds of being the first presenter?

   b) What are the odds of being the second presenter?

   c) What are the odds of being the second-last presenter?

7. For a trade show, exhibit spaces are awarded by lottery. There are 150 spaces, and 200 vendors have applied for a space.

   a) What are the odds of a vendor getting the first space awarded?

   b) After 20 spaces have been awarded, how many vendors are left? What are the odds that a vendor will get the next space?

   c) When there is only one space left, what are the odds of getting that space?

Work With It

1. Assume that the probability of giving birth to a boy or a girl is the same.

   a) Use a tree diagram to display all of the possible combinations for the gender of the children in a family with four children.

   b) What is the probability that a family with four children will have two boys and two girls?

2. A minor-league hockey team is raising money by raffling a number of donated items. You put all 25 of your tickets in the box for an Ice Caps hockey jersey. There are a total of 300 tickets in the box.

   a) Calculate the probability that you will win the jersey. Express your answer as a fraction in lowest terms.

   b) What are the odds that you will win the jersey?

3. A department store offers scratch-and-win tickets to its customers. The contest rules say that 25% of the tickets are winning tickets.

a) What are the odds of getting a winning ticket? Express the answer as a fraction.

b) If the store prints 10 000 tickets, how many winning tickets are there?

c) What is the probability of receiving a winning ticket?

4. Campsites at a popular park are awarded by lottery.

a) This year, the odds of getting a campsite are 1:3. What is the probability of getting a site?

b) The odds against getting a campsite last year were 3:5. What was the probability of getting a site last year?

Discuss It

5. A charity lottery claims that the chance of winning a prize is 1 in 5. You buy 5 tickets. Are you guaranteed to win? Explain why or why not.

6. Dylan says that the odds of drawing a red card from a deck are 50–50. What does he mean?

7. What condition must there be for the odds to be "in your favour"?

8. In what ways are probability and odds the same? In what ways are they different?

Theoretical and Experimental Probability

Jake and Taylor are drawing straws to decide who will clean up the dinner dishes. They are playing the best 2 out of 3. This means that the person who draws the short straw twice has to do the dishes. Jake knows he has a 50% chance of drawing the short straw. However, both times he chose a straw, Jake got the short straw. That means he drew the short straw 100% of the time. Why does the probability not always match the actual results?

Materials

• 2 dice
• grid paper
• ruler

theoretical probability

• a ratio that compares the number of possible successful outcomes to the total number of possible outcomes
• determined by reason or calculation

Explore Theoretical and Experimental Probability

Determine the **theoretical probability** of obtaining each of the possible sums when rolling two dice.

1. In your notebook, copy and complete the following table.

Total of Two Dice	Number of Combinations	Theoretical Probability	
		Fraction of the Total Number of Combinations	**Percent of the Total (to nearest percent)**
2	1		
3	2	$\frac{2}{36} = \frac{1}{18}$	

2. Look at the fractions in your table. Which fractions can be rewritten in lowest terms? In your table, write these fractions in lowest terms. An example has been done for you in step 1.

3. On grid paper, create a bar graph showing your results. Label the *x*-axis "Sum of the Dice." Label the *y*-axis "Percent of the Total (to nearest percent)." Include a title for the graph.

4. Reflect

   a) Two dice are rolled 100 times. How many times should the total of the dice be 2? Explain your answer.

   b) How many times would you expect to roll a total of 8 with 1000 rolls of two dice?

   c) The theoretical probability of rolling a total of 7 with two dice equals the sum of the probabilities of rolling which other numbers?

5. Extend Your Understanding

   a) Roll two dice 50 times. Record the results in a table like this.

Total of Two Dice	Tally	Experimental Probability		
		Number of Times Rolled	Fraction of the Total Number of Rolls (in lowest terms)	Percent of the Total Number of Rolls
2				
3				
4				

   b) How many times did you roll a total of 5?

   c) What is the **experimental probability** of rolling a total of 5, as a percent?

   d) What is the theoretical probability of rolling a total of 5, as a percent? Refer to the table you completed in step 1.

   e) Does the experimental probability you determined in part c) match the theoretical probability you found in part d)?

6. Look at the rest of your experimental results.

   a) How many times did your experimental probability match the theoretical probability?

   b) Explain why the experimental probability might not always match the theoretical probability.

experimental probability

• a ratio that compares the number of times an event occurs to the total number of trials or tests

• determined by experiment

On the Job 1

Understand Probabilities

A gasoline retailer runs a promotion with a 1 in 7 chance of winning a prize with each purchase. Kevin makes a purchase.

a) Calculate Kevin's chance of winning, to the nearest percent.

b) Are the odds of winning in Kevin's favour? Explain.

c) What is the probability of Kevin not winning?

d) How many purchases would guarantee Kevin winning a prize?

Solution

a) The chance of winning is 1 out of 7. Calculate this as a percent:

$1 \div 7 \times 100 = 14.285\ldots$

$$\boxed{C}\ \boxed{1}\ \boxed{\div}\ \boxed{7}\ \boxed{2nd}\ \boxed{\%}\ \boxed{=}\ \boxed{14.28571429}$$

> Your calculator may require a different sequence of keystrokes.

Kevin's chance of winning, to the nearest percent, is 14%.

b) No. The odds are 6 to 1 against Kevin winning.

c) There is more than one method for determining the probability of not winning.
Method 1:
Kevin's probability of winning is 1 in 7. So, his probability of not winning is 6 in 7.
Method 2:
Kevin's probability of winning is about 14%. So, his probability of not winning is about 86%.

d) There is no guarantee of winning. Each purchase is an independent event. There is a 1 in 7 chance of winning this time, and the next time, and the time after that. Kevin could win every time or never win. Chances are, though, that he will win about 1 in 7 times.

Your Turn

The gas retailer improves the probability of winning to 1 in 6.

a) Calculate the probability of Kevin winning, to the nearest percent.

b) Calculate the probability of Kevin not winning, to the nearest percent.

c) Calculate the odds of Kevin winning.

d) Kevin says, "I think I'll win more often than 1 in 6 times!" Do you think that people's expectations affect whether or not they will win?

F.Y.I.

Probabilities are often stated in terms of the chance of winning. Odds are often stated in terms of the likelihood of not winning.

Strategy

Develop Alternative Approaches

Some people prefer to work with percents, and some prefer fractions. Others like to see data in other ways, such as decimal numbers, graphic organizers, or pictures. Which way do you prefer to see data?

Try It

1. You flip a coin once.

  a) What is the probability of getting tails?

  b) What are the odds against getting tails?

  c) You do not get tails. If you flip again, will you get tails?

2. You flip a coin 80 times.

  a) Theoretically, how many tails should you get?

  b) In an actual experiment, how many tails would you get?

3. You roll two dice.

  a) What is the probability of rolling a sum of 5 or less?

  b) What is the probability of rolling a sum that is an even number?

  c) What are the odds of rolling a sum that is an even number?

  d) What are the odds against rolling a sum of 10 or more?

4. A spinner is divided into 10 sections. Write each answer as a fraction and as a percent. State the probability of the spinner

  a) landing on yellow

  b) landing on red

  c) landing on green

  d) landing on blue

  e) landing on yellow or blue

  f) not landing on blue
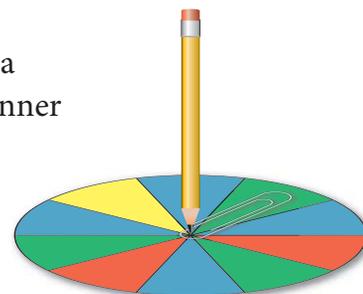
Apply It

5. a) You draw a card from a standard deck of 52 cards. As a fraction, what is the probability of drawing the 7 of clubs?

  b) Express the probability in part a) to the nearest percent.

  c) You draw the 10 of diamonds and do not replace it in the deck. What is the probability of drawing the 7 of clubs on your next try?

  d) Express the probability in part c) to the nearest percent.

  e) How many cards do you need to draw from the deck and not replace before the rounded percent of drawing the 7 of clubs changes to a different percent?

6. A standard deck of 52 cards has four suits: spades, hearts, diamonds, and clubs.

a) You select a card from a deck. What is the probability of drawing a spade?

b) What do you predict the results would be if you repeated the experiment in part a) 40 times?

c) Create a bar graph of what the results in part b) would look like. Label the *x*-axis "Suits" and label the *y*-axis "Number of Draws."

d) Using a standard deck of 52 cards, shuffle the cards and select one card. Record the suit in a frequency table and then replace the card. Do this 40 times.

e) Create a bar graph of your results.

f) Compare your bar graphs in parts c) and e). How close were your results to your prediction?

7. a) Flip a coin 30 times. Record the results in a table.

b) How many times out of 30 did you get tails?

c) Express your answer as a percent.

d) What results would you predict?

e) What percent of your class obtained the results you predicted?

8. You have three dice.

a) What is the total number of outcomes?

b) List all of the combinations of rolling a sum of 7.

c) What is the probability of rolling a sum of 7?

9. Tatum is designing a game in which the theoretical probability of each possible outcome happening is 20%. She wants to create a circular spinner divided into equal sectors. Each sector will have a different colour.

a) How many colours does Tatum need to use for the spinner?

b) Draw or create a spinner that Tatum could include in her game.

10. The game of Yahtzee™ is played with five dice. A "yahtzee" occurs when all five dice show the same number. What is the probability of rolling a yahtzee with one roll of the dice?

On the Job 2

Interpret Probabilities

Jana loves to go out with her friends for coffee. Their favourite coffee shop is having a promotion. A prize is awarded in 1 out of every 9 coffee cups. Jana has won on 3 of her first 4 purchases.

a) What is the theoretical probability of winning, expressed as a percent?

b) Should Jana have won with her first cup? Explain.

c) What about her second cup? Explain.

d) Will Jana win next time?

e) Currently, what percent of the time is she winning?

f) Explain how Jana could win 3 times in a row.

Solution

a) The probability of someone winning is $\frac{1}{9}$, or approximately 11%.

b) No. Theoretically, Jana should not have won, since the probability of winning is only about 11%.

c) No. Theoretically, Jana should not have won, because the odds are always against winning.

d) It is possible, but there is no way to know for certain.

e) Jana has won 3 out of 4 times. She is winning 75% of the time, which is much better than the theoretical probability of winning.

f) Jana was lucky to win 3 out of 4 times. It is likely that if she continued to play again and again, her results would begin to more closely match the theoretical probability of winning.

Your Turn

Jana's friend, Laura, has not had a winning cup in her first 4 purchases.

a) What is the probability of not winning, expressed as a percent?

b) If Laura buys 9 cups of coffee, should she get 1 win? Explain.

c) Will Laura get 1 win in her first 9 cups?

d) Should Laura expect to win next time?

Try It

1. List the following contest winning results from most successful to least successful.

   a) 1 winning outcome in 5 tries

   b) 3 winning outcomes in 13 tries

   c) 6 winning outcomes in 25 tries

   d) 4 winning outcomes in 19 tries

   e) 7 winning outcomes in 33 tries

   f) 5 winning outcomes in 19 tries

2. A department store has a promotion involving scratch-and-win cards. You play 5 cards and win once.

   a) What percent of the time did you win?

   b) You find out that the odds of winning are 1 in 5. What is the probability of winning?

   c) Did you do better or worse than the odds of winning?

3. You flip a coin once and get heads. You flip it a second time and get heads again. What is the probability of getting tails when you flip the coin a third time?

Apply It

4. Lisa and Sara Dougall have both applied for spots in a carpentry apprenticeship program. The spots are awarded by lottery. They find out that half of the students who applied will get a spot.

   a) What are the odds that both of the Dougall sisters will get a spot?

   b) Both Lisa and Sara get a spot. Explain how this is possible.

5. Place each letter of the alphabet on a separate tile or piece of paper. Put the items in a container or bag.

a) As a fraction, what is the probability of drawing a vowel from the container on your first attempt? Consider Y to be a vowel.

b) Express the answer to part a) as a percent.

c) Imagine you select a tile from the container, record the result, and then replace the tile. How many times would you expect to draw a vowel if you selected 26 times?

d) How many times would you expect to draw a vowel if you selected 50 times?

e) Select a tile 50 times. Tally your results in a table like the one shown. Make sure you select a tile and then replace it exactly 50 times.

Vowels	Consonants
|||	++++ ||||

f) How many times did you select a vowel?

g) Express your answer to part f) as a percent.

h) How close were your actual results to your predicted results?

6. Alisha and her family are playing Monopoly™. It is her turn to roll the two dice. She needs to roll 5 or greater to avoid landing at her sister's houses. What is the probability that Alisha will avoid her sister's houses?

7. MINI LAB Work in a group to design a simulation experiment.

STEP 1

Read the following scenario: A submarine sandwich shop is having a promotion. For every sandwich purchased, there is a 1 in 6 chance of winning a prize. Theoretically, how many winners would you get if you ordered 50 sandwiches for a party?

STEP 2

As a group, discuss what roll of two dice has the same probability of occurring as in the scenario in step 1.

STEP 3

Determine how the answer to step 2 could be used as a simulation of the chances of winning in the sandwich shop's promotion.

STEP 4

a) Roll two dice exactly 50 times. Consider the roll you identified in step 2 to be a win. All other sums of the dice are not a win. How many times did you win?

b) Compare your prediction in step 1 to your experimental results.

c) How many students in your class had experimental results better than the theoretical results of the sandwich shop's promotion? Explain why you think this happened.

8. Problems related to machine repair are often the result of nut or bolt malfunction. The main reason for this happening is improper torque. Several car tires were tested after servicing to see if the lug nuts were properly torqued. Out of 95 cars tested, 63 had properly torqued lug nuts.

a) What is the probability that a car is serviced correctly?

b) During one day, 15 cars are brought in for tire service. How many cars do you predict will be serviced correctly?

c) Why might the actual result be different from your prediction?

9. To win most lotteries, you need to correctly pick a certain amount of numbers. An example is a lottery in which 6 numbers are randomly selected from a batch of 49 numbers. After being selected, the numbers are not returned to the batch. A grand prize is awarded for picking all 6 numbers.

a) Expressed as a fraction, what is the probability of the first number drawn being a match to one of your six choices?

b) If the first number drawn matches yours, what is the probability of the second number drawn being a match to one of your five remaining choices?

c) If the second number drawn also matches yours, what is the probability of the third number drawn being a match to one of your four remaining choices?

d) Determine the probability of the fourth, fifth, and sixth numbers also matching your choices.

e) Multiply each of the six fractions together to calculate the probability of winning the grand prize in this lottery.

f) Divide the numerator and the denominator of this fraction by the numerator. Express the probability of winning in the form $\dfrac{1}{\blacksquare}$.

Work With It

1. Jack has all 20 of his sports socks in one drawer. He has the same number of black socks as white socks. The socks are in the drawer individually, not in pairs. Jack is trying to get dressed in the dark.

a) What is the chance of Jack reaching into the drawer and pulling out a black sock?

b) What is the chance of Jack reaching into the drawer and pulling out a white sock?

c) What is the fewest number of socks that Jack has to pull out of the drawer to ensure he has a pair? Explain your reasoning.

2. MINI LAB Investigate the results of rolling two dice.

STEP 1

a) Copy the table shown in your notebook. In the first column, record all of the possible totals you can get when you roll two dice. The first two have been done for you.

Total of Two Dice	Tally	Number of Times Rolled	Percent of Total
2			
3			

b) Roll two dice exactly 50 times. In the second column, keep a tally of the number of times you roll each total.

c) In the third column, write down the total for each tally.

d) Graph your results. Label the *x*-axis "Total of Two Dice." Label the *y*-axis "Percent of Total."

STEP 2

a) If everyone in your class rolled exactly 50 times, what is the total number of rolls for the class?

b) Collect the class's data. Copy and complete the table below. The example in the table represents the total number of 2s rolled for a class of 18 students.

Total of Two Dice	Total Number of Times Rolled	Percent of Class Total
2	2 + 0 + 0 + 1 + 3 + 1 + 2 + 0 + 1 + 2 + 0 + 3 + 1 + 3 + 0 + 2 + 2 + 2 = 25	
3		

c) Graph the class's results. Label the *x*-axis "Total of Two Dice." Label the *y*-axis "Percent of Class Total."

STEP 3

Compare your graph and the class graph. Which graph is closer in shape to the results you would expect? Explain why you think this is.

3. The game of Battleship™ involves two players. Each player places five ships on their own 10×10 grid. The five ships take up a total of 17 places on the grid. Each player must guess the location of the other player's ships. Grid locations are expressed as a letter indicating a column and a number indicating a row. For example, D3 is located where the fourth column and the 3rd row intersect.

a) What is the likelihood of hitting a ship on your first guess?

b) If your first guess is unsuccessful, does the likelihood of success next time increase, decrease, or stay the same? Explain your answer.

c) The aircraft carrier takes up five spaces on the grid. What is the probability of hitting your opponent's aircraft carrier on your first selection?

	A	B	C	D	E	F	G	H	I	J
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

d) The submarine takes up three spaces on the grid. What is the probability of hitting your opponent's submarine on your first selection?

e) What happens to the probability of hitting your opponent's ships with each successive miss? Explain why.

Discuss It

4. Bruce says that the probability of rolling an even-numbered sum with two dice is 50%. Is Bruce correct? Explain why or why not.

5. Rianne says that since games of chance deal with probability, someone who wins consistently at card games is just lucky. Do you agree with her?

6. Rianne also says that someone who often wins at things such as coffee shop promotions is just lucky. Do you agree with her?

7. Which is more likely to occur, flipping five coins at once and having them all land heads up or rolling a 12 with two dice? Explain.

Working With Probability

Focus On . . .

- working with everyday events involving probability
- solving problems that involve probability
- solving problems that involve the need for precision

Building materials are sometimes defective. For example, fence boards can get warped. If you place a large order of boards to build a new fence, you probably will not be able to use some of the boards that you buy.

Explore Teen Sleep Habits

Sleep is food for the brain. Getting enough sleep improves your health, your mood, and your ability to learn. However, it is estimated that an average of 60% of Canadian teens think they do not get enough sleep.

Work in a small group.

1. Conduct a survey of students in your school on whether or not they think they get enough sleep at night.

2. Copy the table below in your notebook. Record students' responses in the table.

Student	Gets Enough Sleep? (Yes or No)
1	
2	

3. Decide when you think you have collected enough data. This may depend on how many students you are able to survey.

4. Reflect

a) What percent of the students you surveyed think they *do not* get enough sleep?

b) How do you think your school compares to the national average? How do you think your community compares to the national average?

5. Extend Your Understanding

a) What is the approximate population of your school? your community?

b) Assuming that your results reflect the population, what is the approximate number of teens who think they do not get enough sleep in your school? in your community?

Web Link

Web Link

For more information about the sleep habits of Canadian teens, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Puzzler

A game-show host asks you to choose among three doors: #1, #2, and #3. Behind one of the doors, there is a new car. Behind the other two doors, there are rocks. The host knows which door hides the car. You choose a door. The host shows you a rock behind one of the two doors that you did not choose. He then gives you the opportunity to change your choice. Assume that the host always does this, no matter which door you guess. Should you change your choice?

On the Job 1

Work With Precision of Probabilities

You work at a factory that makes CFLs (compact fluorescent lights). You analyse a recent batch of a new type of bulb. Your analysis shows that the number of defective light bulbs in the batch is about 1 in 36.

a) Express the defect rate as a percent to five decimal places and to the nearest percent.

b) Your plant makes 1 000 000 of the light bulbs. Using the two defect rates from part a), calculate the potential number of defective light bulbs. What is the difference between the two results?

c) Which degree of precision do you think the manager of the factory would want to see? Explain your reasoning.

Solution

a) $\dfrac{1}{36} = 0.027\ldots$ $\boxed{\text{C}}\ \boxed{1}\ \boxed{\div}\ \boxed{36}\ \boxed{=}\ \boxed{0.0277777778}$

Rounded to five decimal places, the defect rate is 2.77778%.

Rounded to the nearest percent, the defect rate is 3%.

b) $1\ 000\ 000 \times 2.77778\% = 1\ 000\ 000 \times 0.0277778$
$$= 27\ 777.8$$

One million bulbs with a defect rate of 2.77778% should produce about 27 778 defective bulbs.

$1000\ 000 \times 3\% = 1\ 000\ 000 \times 0.03$
$$= 30\ 000$$

A defect rate of 3% should produce 30 000 defective bulbs.

$30\ 000 - 27\ 778 = 2222$

The difference between the potential number of defective bulbs in 1 million bulbs is about 2200 light bulbs.

c) The manager of the factory would likely want the more precise representation of the situation. So, the manager would prefer to know the defect rate of 2.77778%.

Jay is ordering boards from a lumberyard. About 1 in 7 boards at the lumberyard are warped.

a) What percent of the boards are warped? Express your answer to five decimal places and to the nearest percent.

b) Jay orders 300 boards from the lumberyard. Using the two defect rates from part a), calculate the potential number of warped boards Jay will receive. What is the difference between the two results?

c) To which degree of precision do you think Jay would want to know the probability of a board being warped? Explain your reasoning.

Puzzler

A mathematician, her husband, and their teenage daughter all play chess. One day, the daughter asks her mother for $20 to go out on Friday night. Her mother thinks for a moment and replies, "Let's do it this way. Today is Tuesday. You will play a game of chess tonight, tomorrow, and Thursday. If you win two games in a row, you get the money."

"Do I play against you or Dad first?"

"You choose," the mathematician says.

The daughter knows that her mother plays a stronger game than her father. To maximize her chance of winning two games in a row, should she play mother-father-mother or father-mother-father?

Try It

1. What are the odds of obtaining success in the following experiments?

   a) flipping a coin and it lands heads up

   b) rolling one die and the number 3 is facing up

   c) cutting a deck of cards and getting a heart

   d) rolling one die and an odd number is facing up

2. What results would you expect from each experiment?

   a) You flip a coin 50 times.

   b) You roll one die 60 times. How many 3s would you get?

   c) You cut a deck of cards 40 times. How many hearts would you get?

   d) You roll one die 80 times. How many times would you roll an odd number?

3. Calculate each probability as a percent, to five decimal places, and to the nearest percent.

   a) a 1 in 6 chance of a car collision over a 10-year period

   b) a 1 in 11 chance of a hurricane making landfall

   c) a 1 in 9 chance of a certain model of car needing brakes repaired within 3 years

   d) 1 in 10 000 patients experiencing side effects from a certain medical treatment

4. Calculate the success rate of the following results. Express each rate as a fraction in lowest terms.

   a) You want heads. You flip a coin 50 times and get heads 30 times.

   b) You want 3s. You roll a die 60 times and get a 3 five times.

   c) You want hearts. You cut a deck of cards 40 times and get 6 hearts.

   d) You want odd numbers. You roll a die 80 times and get 37 odd numbers.

5. Convert each answer to #4 to a percent. Round each percent to the degree of precision you think is appropriate.

6. Reena and some friends are building a fence in her backyard. Reena determines that she needs 240 fence boards. About 90% of the fence boards at the lumberyard are straight enough to use. How many fence boards should Reena order?

7. A manufacturer of outdoor LED light bulbs claims to have a defect rate of 0.2%.

a) What percent of the bulbs should function properly?

b) In a strand of 50 bulbs, is it likely that all 50 bulbs will work properly? Explain your reasoning.

c) According to the theoretical defect rate, how many strands of 50 bulbs would you need to buy to have one defective bulb?

d) The manufacturer of the bulbs offers a "100% replacement guarantee" to its customers. Discuss with a partner why the manufacturer would have this policy.

8. There are many steps involved in getting the egg from the hen to the refrigerator in a supermarket. Sometimes, eggs break. The manager of a supermarket says that about 1 in 250 eggs breaks.

a) Are you likely to find a broken egg inside a carton of a dozen eggs?

b) About how many dozen eggs would you likely need to inspect before finding a broken one?

c) Nancy says that she always checks the egg carton before buying eggs because she sees "lots of broken ones." Do you think that her claim is likely to be true? Explain why or why not.

9. A 100-year flood prediction is based on the idea that the level of water in any location will reach flood levels, on average, every 100 years. In other words, there is a 1 in 100 chance of there being a flood in any given year. These predictions for flood levels are based on statistical averages.

a) In a certain location, it has been 100 years since a 100-year flood has taken place. Should the citizens of this place prepare themselves for a 100-year flood to occur any day?

b) Research to find out the 100-year flood prediction for your area. When was the last time a 100-year flood occurred in your area?

F.Y.I.

A player with a batting average of .245 hit the ball and got on base 245 times for every 1000 at-bats.

10. A baseball player for the New Brunswick Bears has a batting average of .345.

a) How likely is it that the player will hit the ball and get on base?

b) Why do you think a batting average of .400 or over is very rare? Research to find out how many players in Major League Baseball currently have a batting average of .400 or over.

On the Job 2

Make Decisions Based on Probabilities

The owner of a paving company routinely checks weather forecasts to schedule jobs. Dry, sunny weather is the best for laying down asphalt. Below is a six-day forecast from a weather web site.

	Monday Sept. 13	Tuesday Sept. 14	Wednesday Sept. 15	Thursday Sept. 16	Friday Sept. 17	Saturday Sept. 18
	Cloudy With Sunny Breaks	Rain	Isolated Showers	Mostly Sunny	Sunny	Sunny
P.O.P.	40%	90%	60%	20%	20%	10%
High	18 °C	16 °C	17 °C	18 °C	21 °C	22 °C
Low	11 °C	13 °C	9 °C	14 °C	16 °C	18 °C
24-Hr Rain	close to 1 mm	close to 20 mm	1–3 mm			

a) What does P.O.P. stand for?

b) Explain what P.O.P. means.

c) How can P.O.P. help the owner of the paving company?

d) List the best days for paving during the week shown.

Solution

a) P.O.P. stands for probability of precipitation.

b) P.O.P. is a meteorologist's prediction of the likelihood that there will be some form of precipitation during the day.

c) The ideal conditions for paving are hot and sunny with no precipitation in the forecast. The owner considers the likelihood of rain. The owner may want to look at an hour-by-hour forecast for Tuesday and Wednesday to help in decision making.

d) Tuesday's forecast has a 90% chance of rain and Wednesday has a 60% chance of rain. Tuesday and Wednesday may not be good days. All the other days that week should be fine for paving.

Your Turn

Refer to the six-day forecast above. You have planned an outdoor family picnic for September 18.

a) How would you advise family members about the probability of precipitation?

b) What would you advise family members to wear?

Try It

1. a) Expressed as a percent, what is the probability of something that will happen *for certain*?

b) Expressed as percents, what is the range of probabilities of something that is *likely* to happen?

2. Respond to the statements in parts a) to j) with one of the following:

A It will happen.

B It is likely to happen.

C It is equally likely and unlikely to happen.

D It is not likely to happen.

E It will not happen.

Explain your reasoning in each case.

a) The next person to walk into your classroom will be left-handed.

b) The next person to walk into your classroom will have brown eyes.

c) The sun will rise tomorrow morning.

d) It will snow in Mexico City this afternoon.

e) The next person you speak to on the phone will be female.

f) The next person you text will be male.

g) If you flip four coins at once, they will all land heads up.

h) If you flip two coins at once, they will both land heads up.

i) The moon will be visible tomorrow morning.

j) The Toronto Maple Leafs will win the Stanley Cup in your lifetime.

3. Four fast-food restaurants are having contests. The chance of winning is different for each one.

Top Burgers: 1 in 20 probability of winning
Pizza Perfect: 56 in 60 probability of not winning
Chicken Kwik: 1 to 20 odds of winning
Real Meal: 94 to 6 odds of not winning

Which contest gives you the greatest chance of winning?

F.Y.I.

The average daily high temperature in Mexico City in February and September is 23 °C.

Apply It

4. Below are the dates over a 20-year period when the ice broke in a river.

April 27   April 24   April 28   April 27   May 1

April 26   April 25   April 26   April 22   April 21

April 30   May 2   April 25   April 29   April 24

April 24   April 20   April 22   April 27   April 23

a) What percent of the years did the ice break before April 30?

b) What percent of the years did the ice break before April 25?

c) What percent of the years did the ice break before April 20?

d) What do you think is the probability of the ice breaking after May 2? Explain why.

e) A group of tourists wants to go ice fishing in this river. They would like to fish for the week of April 25 to May 1. What would you advise them about the probability of the river being frozen?

5. A pharmaceutical company is testing a new anti-nausea drug. In one test, they found that 0.2% of the test subjects displayed the same mild side effect.

a) If the drug is tested on a population of 20 000, how many will display the side effect?

b) The company expects more than 10 million users in the first year. If the drug is released to the public now, how many may experience side effects?

c) What factors would you consider in your decision to take or not take the drug?

6. A store that sells ceramic tiles claims that about 1% of the tiles they sell arrive at the store damaged. The tiles are sold in boxes of eight.

a) On average, there is approximately one damaged tile in how many boxes?

b) A customer comes to the store to buy 275 tiles. Predict how many tiles will be damaged. Explain your reasoning.

c) How many tiles should the customer buy? Explain your answer.

7. Ted has the chance to buy a ten-year-old car for $2000 less than the listed price for a car of this year and model. He finds out that, on average, 1 in 7 of these cars needs a new transmission after ten years. The cost of a new transmission is about $2300. Should he buy the car? Explain your reasoning.

Work With It

1. A grocery store receives a shipment of raspberries. On average, 10% of the raspberries in a shipment are spoiled.

   a) Design a simulation experiment for this situation.

   b) Run the experiment using 50 selections, or "trials."

   c) What was your success rate?

   d) Relate your success rate in the experiment to the number of spoiled raspberries in a shipment.

2. Many home entertainment products come with a one-year warranty. An electronics retailer offers a two-year extended warranty plan to all of its customers. It costs $50. About 1.5% of a certain model of TV selling for approximately $800 is known to break down within three years.

   a) Last year the retailer sold 700 of these TVs. About how many TVs will break down in the next three years?

   b) Calculate the value of the sales of the TVs.

   c) The retailer sold the extended warranty with 12% of their TV sales. How many customers purchased the extended warranty?

   d) Why do you think the retailer offers an extended warranty?

   e) Would you purchase the extended warranty? Explain.

3. A basketball player makes 82% of his free throws. He has been successful in his last eight attempts. Do you think his next attempt will be successful? Explain.

4. Vera says that the chances of a six-day weather forecast being accurate are "slim to none."

   a) Explain what she means.

   b) Do you agree with Vera? Why or why not?

5. Our language is full of idioms, some of which deal with probability. "You have a better chance of winning the lottery than winning against me" is an expression that a person might use while playing a game with someone.

   a) What do you think this expression means?

   b) What other expressions dealing with chance or probability have you heard of? Explain each one.

6. The probability of a natural disaster is quite low. However, when a natural disaster occurs, insurance companies have to handle the damage claims that are made. For example, the 1994 hailstorm in Salmon Arm, BC, resulted in a total of $15 million in insured losses. How can insurance companies afford to pay these claims? What role does probability play?

F.Y.I.

An idiom is an expression that is not meant to be taken literally. It is meant to convey something other than the usual meanings of its individual words.

Skill Check

What You Need to Know

Section	After this section, I know how to . . .
1.1	■ understand the difference between accuracy and precision ■ determine the importance of degrees of accuracy and precision ■ calculate maximum and minimum values, using a given degree of tolerance
1.2	■ determine the probability of an event ■ express a probability as a fraction, a decimal, and a percent, and in words ■ determine the odds for and against an event
1.3	■ compare theoretical probability and experimental results ■ express a probability as a fraction, decimal, and percent, and in words ■ determine the probability of an event
1.4	■ work with everyday events involving probability ■ solve problems that involve probability ■ solve problems that involve the need for precision

If you are unsure about any of these questions, review the appropriate section or sections of this chapter.

1.1 Measurement, pages 6–19

1. The thermostat on a hot water tank has a tolerance of ± 1.5. If the thermostat is set at 50° C, what are the maximum and minimum temperatures that the water can be?

2. A carpenter measures the diameter of a dowel. It shows 1 in. The store where he bought it labelled the dowel as $\frac{7}{8}''$.

   a) Which measurement is more precise? Explain.

   b) Give at least two possible reasons why the measurements are different.

1.2 Probability and Odds, pages 20–29

3. a) What are the odds of drawing a diamond from a deck of cards?

   b) What are the odds against drawing the ace of diamonds?

4. You are at a wedding reception that has 200 guests sitting at 25 tables. Guests write their name and table number on a ticket for a door prize. Express each answer as a fraction, a decimal, and a percent, and in words.

   a) What is the probability that you will win the door prize?

   b) What is the probability that the door prize will be won by somebody at your table?

   c) Someone at your table has the winning ticket. What is the probability that you are the winner?

1.3 Theoretical and Experimental Probability, pages 30–41

5. List the contest winning results from least successful to most successful.

   a) 1 winning outcome in 10 tries   **b)**   2 winning outcomes in 18 tries

   c) 5 winning outcomes in 52 tries   **d)**   3 winning outcomes in 29 tries

6. a) Predict the results you would expect from flipping a coin 40 times.

   b) Flip a coin 40 times. Record the results.

   c) Why might you not get the results you predicted?

1.4 Working With Probability, pages 42–53

7. A weather forecast says that the P.O.P. (probability of precipitation) today in Mount Pearl, NL, is 75%.

   a) Are the odds in favour of or against precipitation?

   b) What are the odds in favour of precipitation?

   c) How can you use P.O.P. to help in decision making?

8. A clothing store prints 10 000 scratch-and-save tickets to give to its customers on a special promotion day. The tickets give a percent off of all purchases.

   - 5% of the tickets give 30% off   • 10% of the tickets give 20% off
   - 15% of the tickets give 15% off   • 70% of the tickets give 10% off

   a) How many tickets give 30% off? 10% off?

   b) What percent of the tickets printed offer a discount to the customer?

For #1 to #5, select the best answer.

1. The allowable diameter for a 12-in. pipe is 12 in. $\pm \dfrac{1}{16}$ in. Which size would not be allowed?

   A $11\dfrac{15}{16}$ in.   **B** $11\dfrac{61}{64}$ in.

   C $12\dfrac{1}{32}$ in.   **D** $12\dfrac{5}{64}$ in.

2. What are the odds of correctly guessing the answer to a multiple choice question when there are four options?

   A $4:1$ **B** $3:1$

   C $1:3$ **D** $1:4$

3. What is the probability of drawing a face card from a standard deck of 52 cards?

   A   zero

   B   poor

   C   50-50

   D   good

4. What is the probability of rolling a total of 7 with two dice?

   A   zero

   B   less than other totals

   C   the same as other totals

   D   greater than other totals

5. Mike needs to buy sand to fill a sandbox. The sandbox measures 5 ft long by 4 ft wide by 18 in. deep. He determines he needs to buy 360 cubic feet of sand.

   a)   Is his calculation accurate? Explain.

   b)   Is it important for this calculation to be accurate? Explain.

6. At its closest, Earth is 147 098 300 km from the Sun.

   a) What degree of precision do you think is used to state this distance? Explain.

   b) Round the distance to the nearest million kilometres.

   c) Do you think it is acceptable to state the closest distance between Earth and the Sun as your answer to part b)? Explain.

7. Determine the odds in favour of each outcome if you roll one die.

   a) a number greater than 2

   b) a multiple of 2

   c) not a 5

   d) a number divisible by 3

8. A scratch-and-win lottery card comes with four games. The probability of winning at any game is 1 in 4. Jackie says that since she has lost the first three games, she will win the next one.

   a) Is Jackie certain to win the next game? Explain why or why not.

   b) *Could* she win the next game? Explain why or why not.

9. An employee at a discount store is unpacking boxes of drinking glasses. The employee estimates that 10% are chipped or broken.

   a) The glasses come 12 to a box. Is it likely that a box will contain a damaged glass? Explain.

   b) The store sells about 200 drinking glasses each month. How many cases should they order? Explain your reasoning.

10. If the odds against an event occurring are 6 : 1, what is the probability of the event occurring?

Chapter Project

Design a Game

Project

Designing a game involves more than just having a great idea. The great idea must turn into a game that works and that people want to play.

You are going to be a game designer. Use your imagination to design a game similar to Plinko. Plinko is a game of chance that first appeared on the American TV game show *The Price is Right*.

Here is how it works:
- A contestant drops a disk into a slot at the top of the Plinko board.
- The disk bounces off pegs until it reaches the bottom of the board.
- The disk ends up in one of the slots at the bottom.

Work in a team of two or three students.

1. Design or build a Plinko board with one slot at the top of the board for the disk to enter. The board can be any shape or size, and can be as simple or complex as you like.

2. Calculate the probability or the odds of the disk ending up in each of the slots at the bottom of your board.

3. **a)** How could you alter your Plinko board design to change the probability/odds?

   b) For the new design, what would be the probability or the odds of the disk ending up in each of the slots at the bottom of the board?

Web Link

To play a virtual game of Plinko and relate it to probability, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Nim

Nim is a very old game of strategy that originated in China. There are many variations of the game.

Materials
• 64 toothpicks

Work with a partner.

1. **a)** Put 64 toothpicks in a pile on a table.
   b) When it is your turn, take away either one or two toothpicks.
   c) Your opponent then does the same, removing either one or two toothpicks.
   d) Keep taking turns removing one or two toothpicks.
   e) The player who takes away the last toothpick wins.

2. What strategies did you use to play the game?

3. **a)** What is the probability that a player will choose one toothpick?
   b) What are the odds that a player will choose two toothpicks?
   c) What factors might affect the probability and odds you determined?

4. **a)** Add a variation to the game. Play your new game.
   b) Write three questions about probability or odds for your game.